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The complement metric dimension of particular tree

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Abstract. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The distance between vertices u and v in G is denoted by $d(u,v)$, which serves as the shortest path length from u to v. Let $W = \{w_1,w_2,...,w_k\} \subseteq V(G)$ be an ordered set, and v is a vertex in G. The representation of v with respect to W is an ordered set k-tuple, $r(v|W) = (d(v,w_1),d(v,w_2),...,d(w_k))$. The set W is called a complement resolving set for G if there are two vertices $u,v\in V(G)\backslash W$, such that $r(u|W)=r(v|W)$. A complement basis of G is the complement resolving set containing maximum cardinality. The number of vertices in a complement basis of G is called complement metric dimension of G, which is denoted by $\overline{dim}(G)$. In this paper, we examined complement metric dimension of particular tree graphs such as caterpillar graph ($C_{m,n}$), firecrackers graph ($F_{m,n}$), and banana tree graph ($B_{m,n}$). We got $\overline{dim}(C_{m,n}) = m(n+1)-2$ for $m\geq 1$ and $n\geq 2$, $\overline{dim}(F_{m,n}) = m(n+2)-2$ for $m\geq 1$ and $n\geq 2$, and $\overline{dim}(B_{m,n}) = m(n+1)-1$ if $m\geq 1$ and $n>2$.

1. Introduction

Metric dimension was first mentioned by Harary and Melter at 1976 [4]. Generally, the definition of metric dimension is the minimum cardinality of resolving set on a graph. The idea of resolving sets first appeared in two papers by Slater [16, 17]. He introduced the concept of a resolving set for a connected graph under the term *locating set* and minimum resolving set as *reference set*. Then he called the minimum cardinality of resolving set (reference set) by *location number*. In this paper we use the terminology of Harary and Melter which is metric dimension.

Chartrand, *et al.* [3] defined metric dimension as follows. Let $\boldsymbol{G}$ be a connected graph with vertex set $\boldsymbol{V(G)}$ and edge set $\boldsymbol{E(G)}$. The distance between vertices $\boldsymbol{u}$ and $\boldsymbol{v}$ in $\boldsymbol{G}$, is denoted by $\boldsymbol{d(u,v)}$, which serves as the shortest path length from to $\boldsymbol{u}$ to $\boldsymbol{v}$. Let $\boldsymbol{W = \{w_1, w_2, \dots, w_k\} \subseteq V(G)}$ be an ordered set, and $\boldsymbol{v}$ is a vertex in $\boldsymbol{G}$. The representation of $\boldsymbol{v}$ with respect to $\boldsymbol{W}$ is an ordered set, $\boldsymbol{r(v|W) = (d(v,w_1), d(v,w_2), \dots, d(v,w_k))}$. The set $\boldsymbol{W}$ is called a resolving set for $\boldsymbol{G}$ if each vertex in $\boldsymbol{G}$ has a different representation with respect to $\boldsymbol{W}$. A resolving set containing minimum cardinality is called a basis for $\boldsymbol{G}$. The number of vertices in a basis of $\boldsymbol{G}$ is called metric dimension of $\boldsymbol{G}$, which is denoted by $\boldsymbol{dim(G)}$.

There are many researches on metric dimension of graph. Poisson and Zhang [10] found the lower bound and upper bound for metric dimension of unicyclic graphs. Then Chartrand, *et al.* [3] developed their research by getting the dimensions of tree and unicyclic graphs. They also presented the characterization of graph with certain metric dimension. Then Sebo and Tanner [15] defined the terminology of strong metric dimension on graph which then developed by Rodriguez, *et al.* [13] by getting the strong metric dimension of strong product graphs and Oellerman, *et al.* [8] by getting the strong metric dimension on graphs and digraphs. The paper by Dorota Kuziak, *et al*. [7] also found the strong metric dimension of corona product graphs. The concept of metric dimension also developed by Okamoto, *et al.* [9] into local metric dimension and they found the characterization of graph with certain local metric dimension. Rodriguez, *et al.* [12] also developed this research by getting the local metric dimension of corona product graphs while Iswadi, *et al.* [6] got the metric dimension of corona product graphs. Rodriguez, *et al.* [14] further the research by getting the dimensions of rooted product graphs. Baca, *et al.* [2] got the metric dimension of regular bipartite graph, Ali, *et al.* [1] found the metric dimension of some graph containing the cycle. Furthermore, the similarity of metric dimension and local metric dimension were explored by Susilowati, *et al*. [18]

The concept of complement metric dimension was introduced by Susilowati, *et al*. [19] by developing the concept of metric dimension. If the concept of metric dimension applies a set such that each vertex in the graph can be distinguished by the set, then the opposite concept is that a set ensuring at least two vertices that are regarded the same by the set. Such set is called complement resolving set, and the maximum cardinality of the complement resolving set is called complement metric dimension [19].

Susilowati, *et al*. [19] built a definition of complement metric dimension as follows. Let $\boldsymbol{G}$ be a connected graph containing more than two vertices with vertex set $\boldsymbol{V(G)}$ and edge set $\boldsymbol{E(G)}$. The set $\boldsymbol{S \subseteq V(G)}$ is the complement resolving set of $\boldsymbol{G}$ if there are two vertices $\boldsymbol{u, v \in V(G) \backslash S}$, such that $\boldsymbol{r(u|S) = r(v|S)}$. The complement basis of $\boldsymbol{G}$ is complement resolving set that have maximum cardinality. The number of vertices on the complement basis of $\boldsymbol{G}$ is called the complement metric dimension of $\boldsymbol{G}$, denoted by $\boldsymbol{\overline{dim}(G)}$ [19].

Permana [11] did a research on metric dimension of particular tree graph namely banana tree graph, caterpillar graph, and firecrackers graph. While Susilowati *et al* [19] did a research regarding complement metric dimension of graphs such as path graph, cycle graph, star graph, and complete graph. Furthermore, they also determined complement metric dimension of corona and comb product graphs.

So far the research on complement metric dimension of tree graphs has not been found. Therefore, in this paper we analyze the complement metric dimension on tree graph especially on caterpillar graph, firecrackers graph, and banana tree graph.

Definition 1 [19]. Two distinct vertices u and v of graph G is called twin if u and v have the same neighborhood in $V(G) - \{u, v\}$, then they are called true twin or false twin if u and v are adjacent and twin or u and v are not adjacent and twin, respectively.

The following lemma describe the properties of twin that are discovered by Hernando et al [5]

Lemma 1 [5]. If u and v are twin in graph G, then $d(u, x) = d(v, x)$ for every vertex x in $V(G) - \{u, v\}$.

Observation 1 [19]. The maximum of complement metric dimension of a graph of order n is $n - 2$

2. Complement metric dimension of caterpillar graph

Before we dicuss about caterpillar graph, we must know about the star graph first. A star graph is a graph with one central vertex c that is connected to n pendant vertices. A caterpillar graph obtained by connecting the central vertices of the star graphs in sequence. The connected central vertices in caterpillar graph is called the backbone vertices. If the number of pendant vertices that is connected to a central vertex is the same then the caterpillar graph is called regular caterpillar graph. Otherwise, it is called irregular caterpillar graph.

The complement metric dimension of regular caterpillar graph is presented as below.

Theorem 1. If $C_{m,n}$ is a regular caterpillar graph with m backbone vertices and n pendant vertices, then $\overline{dim}(C_{m,n}) = m(n + 1) - 2$, for $m \geq 1$ and $n \geq 2$.

Proof. Let $V(C_{m,n}) = \{c_i | i = 1,2, \dots, m\} \cup \{a_{ij} | i = 1,2, \dots, m, j = 1,2, \dots, n\}$ and $E(C_{m,n}) = \{c_i c_{i+1} | i = 1,2, \dots, m - 1\} \cup \{c_i a_{ij} | i = 1,2, \dots, m, j = 1,2, \dots, n\}$. If $W = V(C_{m,n}) \setminus \{a_{11}, a_{12}\} =$

$\{c_1, a_{13}, a_{14}, \dots, a_{1n}, c_2, a_{21}, a_{22}, \dots, a_{2n}, \dots, c_m, a_{m1}, a_{m2}, \dots, a_{mn}\}$ then we have $a_{11}, a_{12} \in V(C_{m,n})\backslash W$ such that $r(a_{11}|W) = r(a_{12}|W) = (1,2, \dots 2,2,3,3,3, \dots ,3,3,4,4,4, \dots ,4, \dots , m+1)$. Based on Observation 1, the maximum of complement metric dimension of regular caterpillar graph is $|V(C_{m,n})| - 2 = m(n+1) - 2$. Hence, W is complement basis of $C_{m,n}$ and $\overline{dim}(C_{m,n}) = m(n+1) - 2$. ■

The complement metric dimension of irregular caterpillar graph is presented as follow.

Theorem 2. If $C_{m;n_1,n_2,\dots,n_m}$ is an irregular caterpillar graph then $\overline{dim}(C_{m;n_1,n_2,\dots,n_m}) = (m + \sum_{i=1}^{m} n_i) - 2$, for $m \geq 1$ and at least one $n_k \geq 2$ for $k = 1,2,\dots,m$.

Proof. Let $V(C_{m;n_1,n_2,\dots,n_m}) = \{c_i | i = 1,2, \dots , m\} \cup \{a_{ij} | i = 1,2, \dots , m, j = 1,2, \dots , n_i\}$ and $E(C_{m;n_1,n_2,\dots,n_m}) = \{c_i c_{i+1} | i = 1,2, \dots , m-1\} \cup \{c_i a_{ij} | i = 1,2, \dots , m, j = 1,2, \dots , n_i\}$. Let $n_1 \geq 2$. If $W = V(C_{m;n_1,n_2,\dots,n_m}) \setminus \{a_{11}, a_{12}\} = \{c_1, a_{13}, \dots , a_{1n_1}, c_2, a_{21}, a_{22}, \dots , a_{2n_2}, \dots , c_m, a_{m1}, a_{m2}, \dots, a_{mn_m}\}$ then we have $a_{11}, a_{12} \in V(C_{m;n_1,n_2,\dots,n_m})\backslash W$ such that $r(a_{11}|W) = r(a_{12}|W) = (1,2, \dots 2,2,3,3,3, \dots ,3,3,4,4,4, \dots ,4, \dots , m+1)$. Based on Observation 1, the maximum of complement metric dimension of irregular caterpillar graph is $|V(C_{m;n_1,n_2,\dots,n_m})| - 2 = (m + \sum_{i=1}^{m} n_i) - 2$. Hence, W is complement basis of $C_{m;n_1,n_2,\dots,n_m}$ and $\overline{dim}(C_{m;n_1,n_2,\dots,n_m}) = (m + \sum_{i=1}^{m} n_i) - 2$. ■

The following is the example of complement metric dimension of caterpillar graph.

Example 1. Find the complement metric dimension and complement basis of $C_{3,3}$.

Solution:

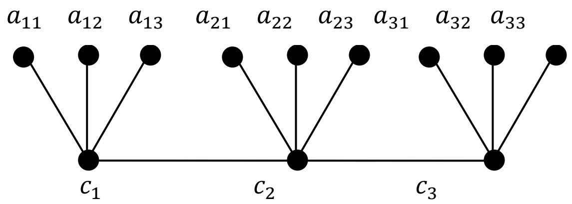

Figure 1. Regular caterpillar graph $C_{3,3}$.

Based on Theorem 1, $\overline{dim}(C_{3,3}) = 3(3+1) - 2 = 10$. Let $W = \{a_{11}, a_{12}, a_{13}, c_1, a_{21}, a_{22}, a_{23}, c_2, a_{31}, c_3\}$. Since $a_{32}, a_{33} \in V(C_{3,3})\backslash W$ and $r(a_{32}|W) = r(a_{33}|W) = (4,4,4,3,3,3,3,2,2,1)$ then W is the complement basis of $C_{3,3}$.

Example 2. Find the complement metric dimension and complement basis of $C_{3;3,2,4}$.

Solution:

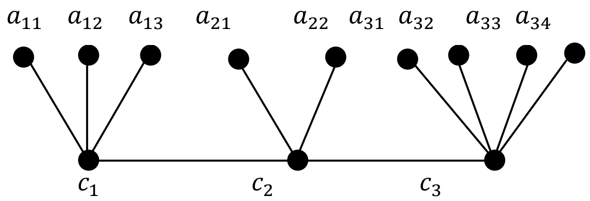

Figure 2. Irregular caterpillar graph $C_{3;3,2,4}$.

Based on Theorem 2, $\overline{dim}(C_{3;3,2,4}) = 3 + 3 + 2 + 4 - 2 = 10$. Let $W = \{a_{11}, a_{12}, a_{13}, c_1, a_{21}, a_{22}, c_2, a_{31}, a_{32}, c_3\}$. Since $a_{33}, a_{34} \in V(C_{3;3,2,4})\backslash W$ and $r(a_{33}|W) = r(a_{34}|W) = (4,4,4,3,3,3,2,2,2,1)$ then W is the complement basis of $C_{3;3,2,4}$.

3. Complement metric dimension of firecrackers graph

Before we dicuss about firecrackers graph, we must know about the path graph first. A path graph of order n (P_n) is a graph connecting vertex v_i to vertex v_{i+1}, where $i = 1,2,\dots,n-1$. A firecrackers graph obtained by taking n copies of star graphs and connecting the central vertex of a star graph to a vertex of P_n. The vertices of P_n is called the backbone vertices, while the other vertices is called pendant vertices. If the number of pendant vertices that is connected to a vertex of P_n is the same then the firecrackers graph is called regular firecrackers graph. Otherwise, it is called irregular firecrackers graph.

The complement metric dimension of regular firecrackers graph is presented as below.

Theorem 3. If $F_{m,n}$ is a regular firecrackers graph with m backbone vertices and n pendant vertices, then $\overline{dim}(F_{m,n}) = m(n+2) - 2$, for $m \geq 1$ and $n \geq 2$.

Proof. Let $V(F_{m,n}) = \{c_i | i = 1,2,\dots,m\} \cup \{a_i | i = 1,2,\dots,m\} \cup \{a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots,n\}$ and $E(F_{m,n}) = \{c_i c_{i+1} | i = 1,2,\dots,m-1\} \cup \{c_i a_i | i = 1,2,\dots,m\} \cup \{a_i a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots, n\}$. If $W = V(F_{m,n}) \setminus \{a_{11}, a_{12}\} = \{c_1, a_1, a_{13}, a_{14}, \dots, a_{1n}, c_2, a_2, a_{21}, a_{22}, \dots, a_{2n}, \dots, c_m, a_m, a_{m1}, a_{m2}, \dots, a_{mn}\}$ then we have $a_{11}, a_{12} \in V(F_{m,n}) \setminus W$ such that $r(a_{11}|W) = r(a_{12}|W) = (2,1,2,\dots,2, 3,4,5,5,\dots,5,4,5,6,6,\dots,6,\dots,m+3)$. Based on Observation 1, the maximum of complement metric dimension of regular firecrackers graph is $|V(F_{m,n})| - 2 = m(n+2) - 2$. Hence, W is complement basis of $F_{m,n}$ and $\overline{dim}(F_{m,n}) = m(n+2) - 2$. ■

The complement metric dimension of irregular firecrackers graph is presented as follow.

Theorem 4. If $F_{m;n_1,n_2,\dots,n_m}$ is an irregular firecrackers graph then $\overline{dim}(F_{m;n_1,n_2,\dots,n_m}) = (2m + \sum_{i=1}^{m} n_i) - 2$, for $m \geq 1$ and at least one $n_k \geq 2$ for $k = 1,2,\dots,m$.

Proof. Let $V(F_{m;n_1,n_2,\dots,n_m}) = \{c_i | i = 1,2,\dots,m\} \cup \{a_i | i = 1,2,\dots,m\} \cup \{a_{ij} | i = 1,2,\dots,m, j = 1,2, \dots, n_i\}$ and $E(F_{m;n_1,n_2,\dots,n_m}) = \{c_i c_{i+1} | i = 1,2,\dots,m-1\} \cup \{c_i a_i | i = 1,2,\dots,m\} \cup \{a_i a_{ij} | i = 1,2,\dots, m, j = 1,2,\dots,n_i\}$. Let $n_1 \geq 2$. If $W = V(F_{m;n_1,n_2,\dots,n_m}) \setminus \{a_{11}, a_{12}\} = \{c_1, a_1, a_{13}, \dots, a_{1n_1}, c_2, a_2, a_{21}, a_{22}, \dots, a_{2n_2}, \dots, c_m, a_m, a_{m1}, a_{m2}, \dots, a_{mn_m}\}$ then we have $a_{11}, a_{12} \in V(F_{m;n_1,n_2,\dots,n_m}) \setminus W$ such that $r(a_{11}|W) = r(a_{12}|W) = (2,1,2,\dots 2,3,4,5,5,\dots,5,4,5,6,6,\dots,6,\dots,m+3)$. Based on Observation 1, the maximum of complement metric dimension of irregular firecrackers graph is $|V(F_{m;n_1,n_2,\dots,n_m})| - 2 = (2m + \sum_{i=1}^{m} n_i) - 2$. Hence, W is complement basis of $F_{m;n_1,n_2,\dots,n_m}$ and $\overline{dim}(F_{m;n_1,n_2,\dots,n_m}) = (2m + \sum_{i=1}^{m} n_i) - 2$. ■

The following is the example of complement metric dimension of firecrackers graph.

Example 3. Find the complement metric dimension and complement basis of $F_{3,3}$.

Solution:

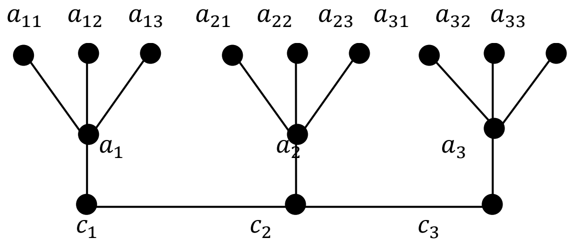

Figure 3. Regular firecrackers graph $F_{3,3}$.

Based on Theorem 3, $\overline{dim}(F_{3,3}) = 3(3+2) - 2 = 13$. Let $W = \{a_{11}, a_{12}, a_{13}, a_1, c_1, a_{21}, a_{22}, a_{23}, a_2, c_2, a_{31}, a_3, c_3\}$. Since $a_{32}, a_{33} \in V(F_{3,3}) \setminus W$ and $r(a_{32}|W) = r(a_{33}|W) = (6,6,6,5,4,5,5,5,4, 3,2,1,2)$ then W is the complement basis of $F_{3,3}$.

Example 4. Find the complement metric dimension and complement basis of $F_{3;3,2,4}$.

Solution:

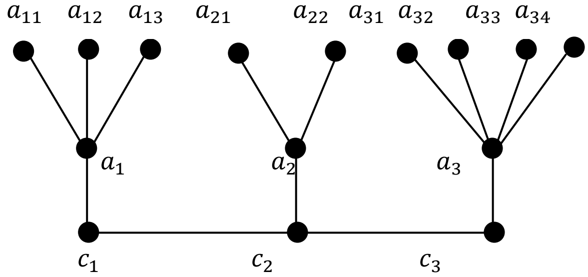

Figure 4. Irregular firecrackers graph $F_{3;3,2,4}$.

Based on Theorem 4, $\overline{dim}(F_{3;3,2,4}) = 2\cdot 3 + 3 + 2 + 4 - 2 = 13$. Let $W = \{a_{11}, a_{12}, a_{13}, a_1, c_1, a_{21}, a_{22}, a_2, c_2, a_{31}, a_{32}, a_3, c_3\}$. Since $a_{33}, a_{34} \in V(F_{3;3,2,4})\backslash W$ and $r(a_{33}|W) = r(a_{34}|W) = (6,6,6,5,4,5,5,4,3,2,2,1,2)$ then W is the complement basis of $F_{3;3,2,4}$.

4. Complement metric dimension of banana tree graph

A banana tree graph is obtained by connecting a pendant vertex of a number of star graphs to a new vertex called root. If the number of pendant vertices of each star graphs is the same then the banana tree graph is called regular banana tree graph. Otherwise, it is called irregular banana tree graph.

The complement metric dimension of regular banana tree graph is presented as below.

Theorem 5. If $B_{m,n}$ is a regular banana tree graph with m star graphs of n pendant vertices, then $\overline{dim}(B_{m,n}) = m(n+1) - 1$, for $m \geq 1$ and $n > 2$.

Proof. Let $V(B_{m,n}) = \{r\} \cup \{c_i | i = 1,2,\dots,m\} \cup \{a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots,n\}$ and $E(B_{m,n}) = \{ra_{in} | i = 1,2,\dots,m\} \cup \{c_i a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots,n\}$. If $W = V(B_{m,n}) \setminus \{a_{11}, a_{12}\} = \{r, c_1, a_{13}, a_{14}, \dots, a_{1n}, c_2, a_{21}, a_{22}, \dots, a_{2n}, \dots, c_m, a_{m1}, a_{m2}, \dots, a_{mn}\}$ then we have $a_{11}, a_{12} \in V(B_{m,n})\backslash W$ such that $r(a_{11}|W) = r(a_{12}|W) = (3,1,2,2,\dots,2,5,6,6,\dots,4,\dots,5,6,6,\dots,4)$. Based on Observation 1, the maximum of complement metric dimension of regular banana tree graph is $|V(B_{m,n})| - 2 = m(n+1) + 1 - 2 = m(n+1) - 1$. Hence, W is complement basis of $B_{m,n}$ and $\overline{dim}(B_{m,n}) = m(n+1) - 1$. ■

The complement metric dimension of irregular banana tree graph is presented as follow.

Theorem 6. If $B_{m;n_1,n_2,\dots,n_m}$ is an irregular banana tree graph then $\overline{dim}(B_{m;n_1,n_2,\dots,n_m}) = (m + \sum_{i=1}^{m} n_i) - 1$, for $m \geq 1$ and at least one $n_k > 2$ for $k = 1,2,\dots,m$.

Proof. Let $V(B_{m;n_1,n_2,\dots,n_m}) = \{r\} \cup \{c_i | i = 1,2,\dots,m\} \cup \{a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots,n_i\}$ and $E(B_{m;n_1,n_2,\dots,n_m}) = \{ra_{in_i} | i = 1,2,\dots,m\} \cup \{c_i a_{ij} | i = 1,2,\dots,m, j = 1,2,\dots,n_i\}$. Let $n_1 > 2$. If $W = V(B_{m;n_1,n_2,\dots,n_m}) \setminus \{a_{11}, a_{12}\} = \{r, c_1, a_{13}, a_{14}, \dots, a_{1n_1}, c_2, a_{21}, a_{22}, \dots, a_{2n_2}, \dots, c_m, a_{m1}, a_{m2}, \dots, a_{mn_m}\}$ then we have $a_{11}, a_{12} \in V(B_{m;n_1,n_2,\dots,n_m})\backslash W$ such that $r(a_{11}|W) = r(a_{12}|W) = (3,1,2,2,\dots,2,5,6,6,\dots,4,\dots,5,6,6,\dots,4)$. Based on Observation 1, the maximum of complement metric dimension of irregular banana tree graph is $|V(B_{m;n_1,n_2,\dots,n_m})| - 2 = (1 + m + \sum_{i=1}^{m} n_i) - 2 = (m + \sum_{i=1}^{m} n_i) - 1$. Hence, W is complement basis of $B_{m;n_1,n_2,\dots,n_m}$ and $\overline{dim}(B_{m;n_1,n_2,\dots,n_m}) = (m + \sum_{i=1}^{m} n_i) - 1$. ■

The following is the example of complement metric dimension of banana tree graph.

Example 5. Find the complement metric dimension and complement basis of $B_{3,3}$.

Solution:

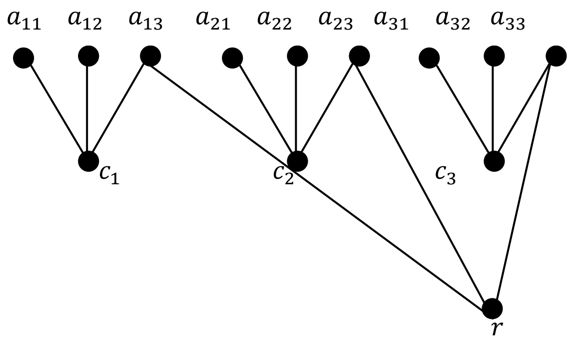

Figure 5. Regular banana tree graph $B_{3,3}$.

Based on Theorem 5, $\overline{dim}(B_{3,3}) = 3(3+1) - 1 = 11$. Let $W = \{r, a_{11}, a_{12}, a_{13}, c_1, a_{21}, a_{22}, a_{23}, c_2, a_{33}, c_3\}$. Since $a_{31}, a_{32} \in V(B_{3,3})\backslash W$ and $r(a_{31}|W) = r(a_{32}|W) = (3,6,6,4,5,6,6,4,5,2,1)$ then W is the complement basis of $B_{3,3}$.

Example 6. Find the complement metric dimension and complement basis of $B_{3;3,2,4}$.
Solution:

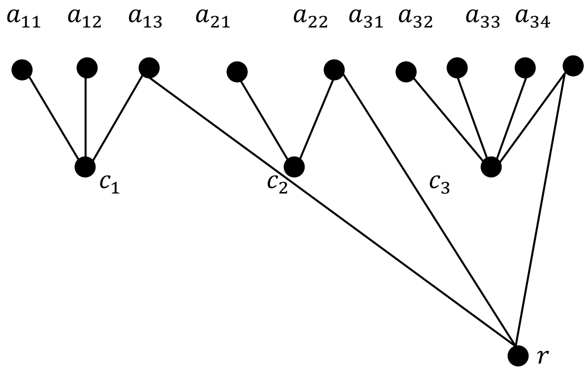

Figure 6. Irregular banana tree graph $B_{3;3,2,4}$.

Based on Theorem 6, $\overline{dim}(B_{3;3,2,4}) = 3 + 3 + 2 + 4 - 1 = 11$. Let $W = \{r, a_{11}, a_{12}, a_{13}, c_1, a_{21}, a_{22}, c_2, a_{33}, a_{34}, c_3\}$. Since $a_{31}, a_{32} \in V(B_{3;3,2,4}) \backslash W$ and $r(a_{31}|W) = r(a_{32}|W) = (3,6,6,4,5,6,4,5,2,2,1)$ then W is the complement basis of $B_{3;3,2,4}$.

5. Conclusion

Based on the discussion in this paper, we have six results on the complement metric dimension of particular tree graphs which are:

1. $\overline{\boldsymbol{dim}}(\boldsymbol{C_{m,n}}) = \boldsymbol{m(n+1)-2}$, where $\boldsymbol{m \geq 1}$ dan $\boldsymbol{n \geq 2}$
2. $\overline{\boldsymbol{dim}}(\boldsymbol{C_{m;n_1,n_2,\dots,n_m}}) = (\boldsymbol{m + \sum_{i=1}^{m} n_i}) - \boldsymbol{2}$ for $\boldsymbol{m \geq 1}$ and at least one $\boldsymbol{n_k \geq 2}$ for $\boldsymbol{k = 1, 2, \dots, m}$.
3. $\overline{\boldsymbol{dim}}(\boldsymbol{F_{m,n}}) = \boldsymbol{m(n+2)-2}$, where $\boldsymbol{m \geq 1}$ dan $\boldsymbol{n \geq 2}$
4. $\overline{\boldsymbol{dim}}(\boldsymbol{F_{m;n_1,n_2,\dots,n_m}}) = (\boldsymbol{2m + \sum_{i=1}^{m} n_i}) - \boldsymbol{2}$ for $\boldsymbol{m \geq 1}$ and at least one $\boldsymbol{n_k \geq 2}$ for $\boldsymbol{k = 1, 2, \dots, m}$.
5. $\overline{\boldsymbol{dim}}(\boldsymbol{B_{m,n}}) = \boldsymbol{m(n+1)-1}$, where $\boldsymbol{m \geq 1}$ dan $\boldsymbol{n > 2}$
6. $\overline{\boldsymbol{dim}}(\boldsymbol{B_{m;n_1,n_2,\dots,n_m}}) = (\boldsymbol{m + \sum_{i=1}^{m} n_i}) - \boldsymbol{1}$ for $\boldsymbol{m \geq 1}$ and at least one $\boldsymbol{n_k > 2}$ for $\boldsymbol{k = 1, 2, \dots, m}$.

From the results above, we can conclude that if a tree of order $\boldsymbol{n}$ has twin vertices, then the complement metric dimension of that tree is $\boldsymbol{n-2}$. This also applies to other graphs that have twin vertices.

The development of complement metric dimension concept is necessary to further this research. Inspired by Sebo and Tanner [15] and Okamoto, *et al.* [9], we can build the concept of strong complement metric dimension or local complement metric dimension in the next research.

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