Properties of cartesian multiplication operations in complete fuzzy graphs, effective fuzzy graphs and complement fuzzy graphs

To cite this article: T Yulianto *et al* 2020 *J. Phys.: Conf. Ser.* **1538** 012022

Properties of cartesian multiplication operations in complete fuzzy graphs, effective fuzzy graphs and complement fuzzy graphs

T Yulianto[1] , N Hayati[1] , I H Agustin[2] , R Amalia[1] , Faisol[1], Kuzairi[1] and B Irwansyah[3]

[1]Mathematics Department, Mathematics and Science Faculty, Islamic University of Madura (UIM) Pamekasan, Indonesia
[2]Mathematics Department, Mathematics and Science Faculty, University of Jember, Indonesia
[3]IAIN Langsa Aceh, Indonesia

E-mail: toniyulianto65@gmail.com

Abstract. This study aims to investigate the properties of cartesian multiplication operations on fuzzy graphs. The properties investigated are cartesian multiplication on complete fuzzy graph, effective fuzzy graph and complement fuzzy graph. The method used in this study is a theoretical research method. The results showed that the Cartesian multiplication of two complete fuzzy graphs is not a complete fuzzy graph. The Cartesian multiplication of the two effective fuzzy graphs is the effective fuzzy graph, while for the Cartesian multiplication of the two fuzzy complement graphs, it is not the same as the Cartesian multiplication of the two effective fuzzy graphs.

1. Introduction

The development of science can never be separated from the role of Mathematics as a tool for various sciences. One of the subjects of mathematics that has applied a lot to date is graph theory [1]. Graph is the simplest mathematical model that can represent the relationship between objects with points representing certain objects and the connecting side between two objects represents the relationship between the two objects. In the case of social networks, points can represent accounts or network users, while the relationships between accounts are represented by sides. If the merits of the relationship are measured by how often the two users communicate, then in describing this relationship a fuzzy graph model design is needed [2].

Fuzzy graphs have been initiated by Rosenfeld in 1975. Fuzzy graphs are an extension of classical graphs that are developed based on concepts in fuzzy logic and fuzzy relations in fuzzy set theory proposed by Zadeh in 1975. If in classical graphs each element (points and side) has a membership value of one or zero, then in fuzzy graphs, each point and side have a membership value located at the close interval [0,1]. The membership value of each of these elements states the degree of membership of the element in a fuzzy graph [2].

Since this fuzzy graph was introduced, researchers began to generalize and develop several studies in classical graphs into fuzzy graphs both in theory and application. modern technology especially in the fields of information theory, neural networks, cluster analysis, medical diagnostics, and control theory. Dey, et. al in 2013 had applied fuzzy graphs to solve traffic lights problems and Swaminathan in 2015 had applied fuzzy graphs to job allocation issues [3]. In theory, Yeh and Bang and Zadeh in 1975 also introduced several concepts related to fuzzy graphs such as the concept of fuzzy connectivity. After the concepts in fuzzy graphs were introduced, more in-depth theoretical results about fuzzy graphs were given, including by Moderson and Peng in 1994 introducing the concepts of operations on fuzzy graphs including joint operations, joins, and compositions on two graphs fuzzy. Furthermore, Sari (2010) defined the Cartesian multiplication operation on the Fuzzy M-Strong graph. In his article, Sari has not reviewed the properties of cartesian multiplication operations on fuzzy graphs related to complete fuzzy graphs, effective fuzzy graphs, and complementary fuzzy graphs [8]. Therefore, based on this description, the purpose of writing this article is to investigate the nature of the Cartesian multiplication operation on two complete fuzzy graphs, and Cartesian multiplication operations on the complement of two effective fuzzy graphs.

2. Preliminary concepts and definitions

2.1. Fuzzy logic

Let U be a set of universes with $x \in U$. A fuzzy set $\tilde{A}$ in U is a set of ordered pairs of elements x with their degree of membership, namely [4,5,6]:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))|x \in U, \mu_{\tilde{A}}(x) = [0,1]\}$$

$\mu_{\tilde{A}}$ is a membership function that maps each $x \in U$ to an interval [0,1]. The value of $\mu_{\tilde{A}}(x)$ in the interval [0,1] is called the membership value or the degree of membership of the element x in $\tilde{A}$, while the interval [0,1] itself is called the membership space. In a strict set, members of the membership space are only zero and one, so the fuzzy set is an extension of a strict set. The degree of membership indicates the amount of involvement of a member in a set.

2.2. Fuzzy graph

Let V be a non-empty and finite set. A fuzzy graph G denoted by $G = (\sigma, \mu)$ is a pair of functions σ expressing the fuzzy set of V and μ is a symmetrical fuzzy relation on σ such that:

(1) $\sigma : V \rightarrow [0,1]$

(2) $\mu : V \times V \rightarrow [0,1]$that fulfills $\mu(v_i, v_j) \leq \min\{\sigma(v_i), \sigma(v_j)\}$, for each $v_i, v_j \in V$

Next, σ is called the fuzzy point set and μ is called the fuzzy side set. The notation $\sigma(v_i)$ in the fuzzy graph represents the degree of membership from the point v_i and $\mu(v_i, v_j)$ expresses the degree of membership from the sides (v_i, v_j). For simple fuzzy graphs, apply $\mu(v_i, v_j) = 0$ for every $v_i, v_j \in V$.

Fuzzy graphs can be represented in an image as a classic graph with points and edges that are equipped with degrees of membership. For each $v_i \in V$. If $\sigma(v_i) = 0$ or $\mu(v_i, v_j) = 0$, then the fuzzy point v_i or the fuzzy side (v_i, v_j) is not drawn [2].

2.3. Basic graph of fuzzy graphs

The basic graph of the fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\sigma^*, \mu^*)$ with σ^*referring to the non-empty set V and $\mu^* \neq E \subseteq V \times V$ Void graph with $\sigma^* = \{v \in V|\sigma(v) > 0\}$ and $\mu^* = \{v_i, v_j \in E|\mu(v_i, v_j) > 0\}$ [2].

2.4. Effective fuzzy graphs

A fuzzy graph $G = (\sigma, \mu)$ is an effective fuzzy graph if it satisfies $\mu(v_i, v_j) = min\{\sigma(v_i), \sigma(v_j)\}$ for each $(v_i, v_j) \in E$ where $E \subseteq V \times V$ [3].

2.5. Completefuzzy graphs

A fuzzy graph $G = (\sigma, \mu)$ is a complete fuzzy graph if it satisfies $\mu(v_i, v_j) = min\{\sigma(v_i), \sigma(v_j)\}$ for each $(v_i, v_j) \in V$.

Based on subsection2.4, every two fuzzy points on a complete fuzzy graph are connected by a fuzzy side, whereas in an effective fuzzy graph not every two fuzzy points are connected by a side. So it can be concluded that the complete fuzzy graph is an effective fuzzy graph but not vice versa [3].

2.6. Complement of fuzzy graphs

According to M.S Sunitha and Vijayakumar, the complement of a fuzzy graph $G:(\sigma, \mu)$ is a fuzzy graph denoted $\bar{G}:(\bar{\sigma}, \bar{\mu})$ where [7]:

i). $\bar{\sigma} = \sigma$ and

ii). $\bar{\mu}(uv) = \sigma(u) \wedge \sigma(v) - \mu(uv) \; \forall u, v \in V$.

2.7. Cartesian multiplication in fuzzy graph

Cartesian multiplication of two fuzzy graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as a fuzzy graph $(\sigma_1 \times \sigma_2, \mu_1 \times \mu_2)$ at $G = (V, E)$ where $V = V_1 \times V_2$ and $E = \{((u, u_2), (u, v_2)) | u \in V_1, (u_2, v_2) \in E_2\}$
$\cup \{(u_1, w), (v_1, w) | (u_1, v_1) \in E_1, w \in V_2\}$

Fuzzy sets $\sigma_1 \times \sigma_2$ and $\mu_1 \times \mu_2$ are defined as [8]:

1) $(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$

2) $(\mu_1 \times \mu_2)((u, u_2), (u, v_2)) = \sigma_1(u) \wedge \mu_2(u_2, v_2), \forall u \in V_1, (u_2, v_2) \in E_2$
$(\mu_1 \times \mu_2)((u_1, w), (v_1, w)) =$
$\mu_1(u_1, v_1) \wedge \sigma_2(w), \forall (u_1, v_1) \in E_1$

In (1), $(\sigma_1 \times \sigma_2)(u_1, u_2)$ is a set of points in a cartesian multiplication operation or that should be written $(\sigma_1 \times \sigma_2)((u_1, u_2))$, but so that it is shorter and clearer then written $(\sigma_1 \times \sigma_2)(u_1, u_2)$.

3. Research method

The research method used in this study is the study of literature on cartesian multiplication operations on fuzzy graphs. Furthermore, this Cartesian multiplication is imposed on complete fuzzy graphs, and the complement of effective fuzzy graphs. The results of the Cartesian Multiplication operation are then set forth as the characteristics of Cartesian Multiplication operations which are written in the form of systematic evidence.

4. Result and discussion

For example, given two complete fuzzy graphs $G_1 = (\sigma_1, \mu_1)$and $G_2 = (\sigma_2, \mu_2)$. The sides of the fuzzy graph of the cartesian multiplication graph fuzzyG_1 and $G_2$are defined as in Definition 7, namely:

$$E = \{((u, u_2), (u, v_2)) | u \in V_1, (u_2, v_2) \in E_2\}$$
$$\cup \{(u_1, w), (v_1, w) | (u_1, v_1) \in E_1, w \in V_2\}$$

As a result the degree of membership of the side of$(\mu_1 \times \mu_2)((u_1, u_2), (v_1, v_2)) = 0$for $u_1 \neq v_1$and$u_2 \neq v_2$.So that not every two points on $G_1 \times G_2$ are connected by a side. Based on Definition 4, the results of the Cartesian multiplication operationG_1 and $G_2$are not complete fuzzy graphs.

4.1. Analyze of effective fuzzy graphs

We have theorem that "*If $G_1 \times G_2$ is an effective fuzzy graph, then G_1 or G_2 is an effective fuzzy graph.*", so for getting proof we can make example. IfG_1 and G_2 are not effective fuzzy graphs, it will be proven that $G_1 \times G_2$ is not an effective fuzzy graph.

Because G_1 and G_2 are not effective fuzzy graphs, then:

$$\mu_1(u_1, v_1) < \sigma_1(u_1) \wedge \sigma_1(v_1) \quad (1)$$
$$\mu_2(u_2, v_2) < \sigma_2(u_2) \wedge \sigma_2(v_2) \quad (2)$$

without eliminating the general nature, then from inequality (1) and (2) it can be assumed that $\mu_2(u_2, v_2) \leq \mu_1(u_1, v_1) < \sigma_1(u_1) \wedge \sigma_1(v_1) \leq \sigma_1(u_1)$ for $((u_1, u_2), (u_1, v_2)) \in E$,where E is defined as in Definition 2.6, which is:$E = \{((u, u_2), (u, v_2)) | u \in V_1, (u_2, v_2) \in E_2\}$

$$\cup \{(u_1, w), (v_1, w) | (u_1, v_1) \in E_1, w \in V_2\}$$
$$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \sigma_1(u_1) \wedge \mu_2(u_2, v_2)$$

In accordance with the definition of Cartesian multiplication and inequality (1) and (2) obtained:$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \sigma_1(u_1) \wedge \mu_2(u_2, v_2)$

$$< \sigma_1(u_1)\sigma_2(u_2) \wedge \sigma_2(v_2)$$

and

$$(\sigma_1 \times \sigma_2)(u_1, u_2) = \sigma_1(u_1) \wedge \sigma_2(u_2)$$
$$(\sigma_1 \times \sigma_2)(u_1, v_2) = \sigma_1(u_1) \wedge \sigma_2(v_2)$$

then

$$(\sigma_1 \times \sigma_2)(u_1, u_2) \wedge (\sigma_1 \times \sigma_2)(u_1, v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_1(u_1) \wedge \sigma_2(v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$
$$= \sigma_1(u_1) \wedge \sigma_2(u_2) \wedge \sigma_2(v_2)$$

So obtained

$$(\mu_1 \times \mu_2)\big((u_1, u_2), (u_1, v_2)\big) < \sigma_1(u_1)\sigma_2(u_2) \wedge \sigma_2(v_2)$$
$$= (\sigma_1 \times \sigma_2)(u_1, u_2) \wedge (\sigma_1 \times \sigma_2)(u_1, v_2)$$

From this proof, it is obtained that $G_1 \times G_2$ is not an effective fuzzy graph, so a contradiction is obtained. So, if $G_1 \times G_2$ is an effective fuzzy graph, then G_1 or G_2 is an effective fuzzy graph.

4.2. Analyze of fuzzy complement graphs

We have theorem that "*For example given an effective fuzzy graph G_1 and an effective fuzzy graph G_2. If G_1^c and G_2^c are fuzzy complement graphs of G_1 and G_2 then $G_1^c \times G_2^c \neq G_1 \times G_2$.*", so for getting proof we can make example. If given two non-empty sets V_1 and V_2. Next are given two effective fuzzy graphs $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$. The graph$G_1^c = (\sigma_1^c, \mu_1^c)$ is the complement of the graph $G_1 = (\sigma_1, \mu_1)$. The graph $G_2^c = (\sigma_2^c, \mu_2^c)$ is the complement of graph $G_2 = (\sigma_2, \mu_2)$.

Based on subsection 2.6, G_1^c has a degree of membership point $\sigma_1^c(u) = \sigma_1(u)$ and sides $\mu_1^c(u_i, u_k) = min\{\sigma_1^c(u_i), \sigma_1^c(u_k)\} - \mu_1(u_i, u_k)$. This also applies to G_2^c having a degree of membership point $\sigma_2^c(v) = \sigma_2(v)$ and sides $\mu_2^c(v_i, v_k) = min\{\sigma_2^c(v_i), \sigma_2^c(v_k)\} - \mu_2(v_i, v_k)$.

Since $G_1 = (\sigma_1, \mu_1)$ is an effective fuzzy graph, it can be said if $\mu_1(u_i, u_k) = min\{\sigma_1(u_i), \sigma_1(u_k)\}$.

This means $\mu_1^c(u_i, u_k) = min\{\sigma_1(u_i), \sigma_1(u_k)\} - min\{\sigma_1(u_i), \sigma_1(u_k)\} = 0$ in other words $(u_i, u_k) \notin E_1^c$. Similarly, because $G_2 = (\sigma_2, \mu_2)$ is an effective fuzzy graph, it can be said if $(v_i, v_k) \in E_2$ then $\mu_2(v_i, v_k) = min\{\sigma_2(v_i), \sigma_2(v_k)\}$.

This means $\mu_2^c(v_i, v_k) = min\{\sigma_2(v_i), \sigma_2(v_k)\} - min\{\sigma_2(v_i), \sigma_2(v_k)\} = 0$, in other words $(v_i, v_k) \notin E_2^c$. Then according to subsection2.7, the Cartesian product of G_1^c and G_2^c is denoted by $G_1^c \times G_2^c = (\sigma_1^c \times \sigma_2^c, \mu_1^c \times \mu_2^c)$ is a function pair with :

$$(\sigma_1^c \times \sigma_2^c)(u, v) = \sigma_1^c(u) \wedge \sigma_2^c(v)$$
$$= \sigma_1(u) \wedge \sigma_2(v)$$
$$= (\sigma_1 \times \sigma_2)(u, v) \tag{3}$$

$$(\mu_1^c \times \mu_2^c)\big((u, v_i), (u, v_k)\big) = \sigma_1^c(u) \wedge \mu_2^c(v_i, v_k)$$
$$= \sigma_1^c(u) \wedge 0$$
$$= 0 \tag{4}$$

$$(\mu_1^c \times \mu_2^c)\big((u_i, v), (u_k, v)\big) = \mu_1^c(u_i, u_k) \wedge \sigma_2^c(v)$$
$$= 0 \wedge \sigma_2^c(v)$$
$$= 0 \tag{5}$$

As a result of equations (3) to (5), it is found $(\sigma_1^c \times \sigma_2^c)(u, v) = (\sigma_1 \times \sigma_2)(u, v)$ but $(\mu_1^c \times \mu_2^c)\big((u, v_i), (u, v_k)\big) = 0$ and $(\mu_1^c \times \mu_2^c)\big((u_i, v), (u_k, v)\big) = 0$. Thus it is proven that $G_1^c \times G_2^c \neq G_1 \times G_2$.

5. Conclusion

Based on the discussion that has been explained in the Thesis with the title "Properties of Cartesian Multiplication Operations in Complete Fuzzy Graphs, Effective Fuzzy Graphs and Compressed Fuzzy Graphs" it is obtained that:

1. Cartesian multiplication results from two complete fuzzy graphs are not complete fuzzy graphs.
2. The results of cartesian multiplication in the complement of two effective fuzzy graphs are not the same as the cartesian multiplication of the two effective fuzzy graphs.

Properties of Cartesian multiplication operations in two fuzzy graphs can also be associated with other operations, so it is necessary to investigate the relationship of cartesian multiplication operations with other operations such as modular, tensor, normal, composition, homomorphic multiplication in two fuzzy graphs. Also, cartesian multiplication operations can be developed from n fuzzy graphs. Do the properties of the Cartesian multiplication operation on two fuzzy graphs also apply to Cartesian multiplication of n fuzzy graphs?.

Acknowledgments
Thanks to DRPM Research and Technology Dikti who have allocated research funds that can then be utilized in the success of this research from research grant in 2019 with contract numbers 263/V/2019. In addition, the authors also thank to Airlangga university, LP2M UIM and Mathematics of UIM who has provided support in the form of initial assistance to conduct research.

References
[1] S. R. Nurshiami, . Suroto and F. Hoeruddin, "Pelabelan Fuzzy Pada Graf," *JMP* , vol. 6, no. 1, pp. 1-12, Juni 2014.

[2] Triyani, C. Baskoro, N. Larasati and A. Wardayani, "Sifat-sifat Operasi Perkalian Modular pada Graf Fuzzy," *Seminar Matematika dan Pendidikan Matematika UNY*, pp. PT-13-PT-18, 2017.

[3] Triyani, N. Larasati and A. Wardayani, "Sifat-sifat Operasi Perkalian Modular pada Graf Fuzzy Lengkap, Graf Fuzzy Efektif dan Graf Fuzzy Komplemen," *Jurnal Matematika Integratif*, vol. 14, no. 1, pp. 31-38, 2018.

[4] T. Yulianto, S. Komariyah and N. Ulfaniyah, "Application of Fuzzy Inference System by Sugeno Method on Estimating of Salt Production," *AIP Conference Proceedings*, vol. 1867, no. 1, pp. 0200391-0200397, 2017.

[5] M. R. Salman, "Konsep dan Sifat Dasar Graf Fuzzy," in *Skripsi*, Makassar, Universitas Negeri Makassar, 2017, pp. 13-14.

[6] S. Kusumadewi, S. Hartati, A. Harjoko and R. Wardoyo, Fuzzy Multi-Attribute Decision Making (Fuzzy MADM), 1 ed., Yogyakarta: Graha Ilmu, 2006.

[7] L. Ratnasari, Y. Sumanto and T. A. Novia, "Komplemen Graf Fuzzy," *Prosiding*, pp. 24-28, 2009.

[8] R. J. Sari, "Graf Fuzzy M-Strong," in *Skripsi*, Semarang, Universitas Diponegoro, 2010, pp. 21-31.