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Abstract. Regression modelling is one Statistical methods that be used to investigate the relationship between predictor variable and response variable. In regression modelling can be estimated with three approaches, such as parametric, nonparametric and semiparametric regression. In this case, we concentrated to elaborate semiparametric regression. Semiparametric regression consists of parametric and nonparametric component. This research examined semiparametric regression model with Fourier series estimator for longitudinal data. By minimizing Weighted Least Square (WLS), the Fourier series estimator depends on the oscillation parameter. The result is the estimator for parameter and curve regression, that be used to model with real data. The optimal model is selected based on minimum Generalized Cross Validation (GCV) which affects the small value of Mean Square Error (MSE) and high determination coefficient so that the model can be used further as estimation and prediction.

1. Introduction

The data model that is often used to determine the pattern of the relationship between the response variables and predictors is the regression model [1]. There are three methods for estimating the regression curve, such as parametric, nonparametric, and semiparametric regression. The combination of parametric regression and nonparametric regression is semiparametric regression [2]. Parametric regression assumes that the regression model is known based on the relationship between the response variable and the predictor variable. Nonparametric regression is assumed to be unknown in the shape of the curve because the estimated shape of the regression curve is not influenced by the subjectivity factor of the researcher [3]. Some estimator for nonparametric regression curves include local polynomial [1], spline [2], local linear [4], kernel [5] and Fourier series [6]. The estimator is also used in semiparametric regression, including semiparametric regression estimators such as spline [7], local linear [8], Fourier series [9], [10], [11]. Most of these estimators work cross section data case. Based on the development of data analysis in regression modeling, there are also other types of data, there are longitudinal data

Longitudinal data is data that is observed repeatedly for each subject in several subjects taken. Longitudinal data assumes that each subject does not depend on each other but between observations in the same subject are interdependent so that there is a correlation. Longitudinal data have the advantage of being in the same number of subjects, the results of error measurements produce an estimator of the effect of a more efficient treatment because longitudinal data are estimated for each observation and are more powerful even if only using fewer subjects [12, 13].

There are several approaches to estimating semiparametric regression curves, one of which is using a Fourier series estimator. One of the advantages of the Fourier series estimator is that it is able to overcome data patterns that have a trigonometric function, in this case the sine function and the cosines function [9]. Fourier series estimators can be obtained by optimizing WLS while to estimate the optimal bandwidth parameters using the GCV method. This research discussed about weighted matrix selection is used Fourier series estimator in semiparametric regression modeling for longitudinal data.

2. Fourier series estimator in semiparametric regression for longitudinal data with Weighted Least Square (WLS)

In this study it provides a representation of the longitudinal data structure in accordance with the Fourier series estimator in semiparametric regression. The data pattern is assumed to be smooth when approached by the Fourier series estimator by determining the optimal smoothing parameters based on GCV optimization. Repeated observations of data for each independent subject are part of the longitudinal data. The relationship between subjects in the longitudinal data model is assumed to be independent from one subject to another, so there is a correlation if observations between subjects are interdependent [12]. Studying how the observed subject changes over time is the main objective of the longitudinal data model. The combination of cross section data and time series data will become longitudinal data. Longitudinal data structures in this study are given in Table 1.

Table 1. The structure of longitudinal data.

Subject	Response	Predictors							
		Parametric				Nonparametric			
	y_{ij}	x_{ij1}	x_{ij2}	...	x_{ijp}	t_{ij1}	t_{ij2}	...	t_{ijq}
1st Subject	y_{11}	x_{111}	x_{112}	...	x_{11p}	t_{111}	t_{112}	...	t_{11q}
	y_{12}	x_{121}	x_{122}	...	x_{12p}	t_{121}	t_{122}	...	t_{12q}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	$y_{1.n}$	$x_{1.n.1}$	$x_{1.n.2}$	...	$x_{1.n.p}$	$t_{1.n.1}$	$t_{1.n.2}$	...	$t_{1.n.q}$
2nd Subject	y_{21}	x_{211}	x_{212}	...	x_{21p}	t_{211}	t_{212}	...	t_{21q}
	y_{22}	x_{221}	x_{22}	...	x_{22p}	t_{221}	t_{222}	...	t_{22q}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	$y_{2.n}$	$x_{2.n.1}$	$x_{2.n.2}$	...	$x_{2.n.p}$	$t_{2.n.1}$	$t_{2.n.2}$	...	$t_{2.n.q}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
nnd Subject	y_{m1}	x_{m11}	x_{m12}	...	x_{m1p}	t_{m11}	t_{m12}	...	t_{m1q}
	y_{m2}	x_{m21}	x_{m22}	...	x_{m2p}	t_{m21}	t_{m22}	...	t_{m2q}
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	$y_{m.n}$	$x_{m.n.1}$	$x_{m.n.2}$	...	$x_{m.n.p}$	$t_{m.n.1}$	$t_{m.n.2}$	...	$t_{m.n.q}$

Observations of the longitudinal data model are generally carried out on n independent subjects with each subject being observed repeatedly (repeated measurements) at different times. If x_{ijp} stating the i^{th} subject at the p^{th} time observation and for the j^{th} predictor, stating the i^{th} subject at the q^{th} time observation and for the j^{th} predictor and stating the response variable measured longitudinally on the i^{th} subject and the j^{th} predictor, then given longitudinal data

$$\left(x_{ijp}, t_{ijq}, y_{ij}\right),\ i=1,2,\ldots,n,\ j=1,2,\ldots,m,\ p=1,2,\ldots,P,\ q=1,2,\ldots,Q \tag{1}$$

The semiparametric regression model for longitudinal data is given as follows:

$$y_{ij}=\beta_{0i}+\sum_{j=1}^{P}\beta_{pi}x_{pij}+\sum_{q=1}^{Q}g_q\left(t_{qij}\right)+\varepsilon_{ij},\ \varepsilon_{i,j}\sim IIN(0,\sigma^2) \tag{2}$$

In equation (2) the semiparametric regression approach used is a combination of a parametric regression model with a known pattern, in this case a linear function with a predictor variable x as many as p, and a nonparametric regression model whose pattern is unknown with the predictor variable t as many as q . y is the notation of the response variable. β is the notation of the parametric regression coefficient. $g_q(t_{qij})$ is the notation of the nonparametric regression curve, the number of observations is denoted by j, ε_{ij} is a random error that identical independently and normally distributed with mean 0, and variance σ^2 [13]. Regression curve in (1) approached by Fourier series estimator. Fourier series is a function of trigonometric polynomials which has a high flexibility. Fourier series is a curve that shows the sines and cosines functions. If given $g(t)$ is a function that can be integrated and differentiable in intervals $[a, a+2L]$, then the Fourier series representation of the interval associated with $g(t)$ containing the cosines Fourier series based on Bilodeau, 1992 [14]. Given data $\left(x_1, x_2, \ldots, x_p, t_1, t_2, \ldots, t_q, \text{and } y_i\right)$, the connection between x_p and y_i considered to follow the parametric regression model, and the connection between t_q and y_i considered to follow a nonparametric regression model, so the semiparametric regression estimator based on Fourier series for longitudinal data as follows :

$$y_{ij}=\beta_{0i}+\sum_{p=1}^{P}\beta_{pi}x_{pij}+\sum_{q=1}^{Q}(b_{qi}t_{qij}+\frac{a_{0qi}}{2}+\sum_{k=1}^{K}(a_{kqi}\cos kt_{qij}))+\varepsilon_{ij},\ \varepsilon_{i,j}\sim IIN(0,\sigma^2) \tag{3}$$

with $\beta_{0i}, \beta_{1i}, \ldots, \beta_{pi}$ are parametric regression coefficients that will be estimated, $b_{1i}, b_{2i}, \ldots, b_{qi}, a_{01i}, a_{02i}, \ldots, a_{0qi}, a_{k1i}, a_{k2i}, \ldots, a_{kqi}$ are parameter in nonparametric regression that the value can be derived based on Weighted Least Square (WLS) optimization result, k is a number that related to the number of oscillation parameter. ε_{ij} is a random error, and ε_{ij} has independent and identically distributed.

Equation (3) can be formed as matrices equation as follows

$$\mathbf{y}=\mathbf{X\beta}+\mathbf{S\eta}+\boldsymbol{\varepsilon},\ \varepsilon_{i,j}\sim IIN(0,\sigma^2) \tag{4}$$

with

$$y=\left(y_{11}, y_{12}, \ldots, y_{1m}, \ldots, y_{n1}, y_{n2}, \ldots, y_{nm}\right)^T$$

$$\beta=\left(\beta_{01}, \beta_{11}, \ldots, \beta_{p1}, \ldots, \beta_{0n}, \beta_{1n}, \ldots, \beta_{pn}\right)^T, \varepsilon=\left(\varepsilon_{11}, \varepsilon_{12}, \ldots, \varepsilon_{1m}, \ldots, \varepsilon_{n1}, \varepsilon_{n2}, \ldots \varepsilon_{nm}\right)^T$$

$$\eta=\left(b_{11}, a_{011}/2, a_{111}, \ldots, a_{k11}, \ldots, b_{q1}, a_{0q1}/2, a_{1q1}, \ldots, a_{kq1}, \ldots, b_{1n}, a_{01n}/2, a_{11n}, \ldots, a_{k1n}, \ldots, b_{qn}, a_{0qn}/2, a_{1qn}, \ldots, a_{kqn}\right)^T$$

$$X=\begin{pmatrix} 1 & x_{111} & x_{211} & \cdots & x_{p11} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 1 & x_{112} & x_{212} & \cdots & x_{p12} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{11m} & x_{21m} & \cdots & x_{p1m} & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{1n1} & x_{2n1} & \cdots & x_{pn1} \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{1n2} & x_{2n2} & \cdots & x_{pn2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{1nm} & x_{2nm} & \cdots & x_{pnm} \end{pmatrix}$$

$$S=\begin{bmatrix} t_{111} & 1 & \cos t_{111} & \cdots & \cos kt_{111} & \cdots & t_{q11} & 1 & \cos t_{q11} & \cdots & \cos kt_{q11} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ t_{112} & 1 & \cos t_{112} & \cdots & \cos kt_{112} & \cdots & t_{q12} & 1 & \cos t_{q12} & \cdots & \cos kt_{q12} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ t_{11m} & 1 & \cos t_{11m} & \cdots & \cos kt_{11m} & \cdots & t_{q1m} & 1 & \cos t_{q1m} & \cdots & \cos kt_{q1m} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & t_{1n1} & 1 & \cos t_{1n1} & \cdots & \cos kt_{1n1} & \cdots & t_{qn1} & 1 & \cos t_{qn1} & \cdots & \cos kt_{qn1} \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & t_{1n2} & 1 & \cos t_{1n2} & \cdots & \cos kt_{1n2} & \cdots & t_{qn1} & 1 & \cos t_{qn2} & \cdots & \cos kt_{qn2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & t_{1nm} & 1 & \cos t_{1nm} & \cdots & \cos kt_{1nm} & \cdots & t_{qn1} & 1 & \cos t_{qnm} & \cdots & \cos kt_{qnm} \end{bmatrix}$$

The estimator for regression curve, and parameter for parametric and nonparametric component can be solved based on WLS optimization. This optimization is done to minimize the goodness of fit for estimator in this research. Theorem 1 give the procedure to get curve estimator based on Fourier series in semiparametric regression for longitudinal data generally. Theorem 2 give the procedure to get parameter estimator for parametric, nonparametric, and semiparametric component with form regression equation as matrix vector equation.

Theorem 1 If a semiparametric regression model is given with a regression curve approximated by the Fourier series as in equation (3), this case for longitudinal data, then the related estimator is as follows:

$$\hat{y}_{ij}=\hat{\beta}_{0i}+\sum_{p=1}^{P}\hat{\beta}_{pi}x_{pij}+\sum_{q=1}^{Q}(\hat{b}_{qi}t_{qij}+\frac{\hat{a}_{0qi}}{2}+\sum_{k=1}^{K}(\hat{a}_{kqi}\cos kt_{qij})) \tag{5}$$

Proof. Semiparametric regression equations for longitudinal data in equation (3) are given which are ε normally distributed with mean 0 and variance σ^2

$$E(y_{ij})=E\left(\beta_{0i}+\sum_{p=1}^{P}\beta_{pi}x_{pij}+\sum_{q=1}^{Q}(b_{qi}t_{qij}+\frac{a_{0qi}}{2}+\sum_{k=1}^{K}(a_{kqi}\cos kt_{qij}))+\varepsilon_{ij}\right)$$

$$E\left(y_{ij}\right)=E\left(\beta_{0i}+\sum_{p=1}^{P}\beta_{pi}x_{pij}+\sum_{q=1}^{Q}(b_{qi}t_{qij}+\frac{a_{0qi}}{2}+\sum_{k=1}^{K}(a_{kqi}\cos kt_{qij}))\right)+E\left(\varepsilon_{ij}\right)$$

$$E\left(y_{ij}\right)=E\left(\beta_{0i}+\sum_{p=1}^{P}\beta_{pi}x_{pij}+\sum_{q=1}^{Q}(b_{qi}t_{qij}+\frac{a_{0qi}}{2}+\sum_{k=1}^{K}(a_{kqi}\cos kt_{qij}))\right)+0$$

So obtained

$$\hat{y}_{ij}=\hat{\beta}_{0i}+\sum_{p=1}^{P}\hat{\beta}_{pi}x_{pij}+\sum_{q=1}^{Q}(\hat{b}_{qi}t_{qij}+\frac{\hat{a}_{0qi}}{2}+\sum_{k=1}^{K}(\hat{a}_{kqi}\cos kt_{qij}))$$

Thus theorem 1 is proven.

Theorem 2 If the semiparametric regression model in longitudinal data approximated by the Fourier series estimator is presented asequation (4) as follows:

i. Estimators for parametric component parameters that do not contain other parameters are as follows:

$$\begin{aligned}\hat{\boldsymbol{\beta}} &= \mathbf{M}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y} - \mathbf{M}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{y} \\ &= \mathbf{M}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left\{\mathbf{X}^{\mathrm{T}} - \mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathrm{T}}\right\}\mathbf{W}\mathbf{y}\end{aligned} \tag{6}$$

ii. Estimators for nonparametric component parameters that do not contain other parameters are as follows:

$$\begin{aligned}\hat{\boldsymbol{\eta}} &= \mathbf{N}\left(\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{y} - \mathbf{N}\left(\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y} \\ &= \mathbf{N}\left(\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\right)^{-1}\left\{\mathbf{S}^{\mathrm{T}} - \mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{X}\left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\right\}\mathbf{W}\mathbf{y}\end{aligned} \tag{7}$$

iii. Estimators for semiparametric regression curves in vector equations are as follows

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{S}\hat{\boldsymbol{\eta}} \tag{8}$$

Proof. Parameter estimation in semiparametric regression model based on Fourier series for longitudinal data obtained by optimization of Weighted Least Square (WLS) is done to minimize the goodness of fit of the semiparametric regression model with the Fourier series approach for longitudinal data

$$\min_{f\in C(0,\pi)}[R(f)] = \min_{f\in C(0,\pi)} \boldsymbol{\varepsilon}^{\mathrm{T}}\mathbf{W}\boldsymbol{\varepsilon} = \min_{f\in C(0,\pi)}\{(\mathbf{y}-\mathbf{f}(\mathbf{x},\mathbf{t}))^{T}W(\mathbf{y}-\mathbf{f}(\mathbf{x},\mathbf{t}))\} \tag{9}$$

$\mathbf{W}$ will be discussed next sectionby elaborating on equation (9), WLS optimization from is given as follows:

$$\begin{aligned}R(\boldsymbol{\beta},\boldsymbol{\eta}) &= (\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta})^{\mathrm{T}}\mathbf{W}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta}) \\ &= (\mathbf{y}^{\mathrm{T}}-\boldsymbol{\beta}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}-\boldsymbol{\eta}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}})\mathbf{W}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta}) \\ &= (\mathbf{y}^{\mathrm{T}}\mathbf{W}-\boldsymbol{\beta}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{W}-\boldsymbol{\eta}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}\mathbf{W})(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta}) \\ &= \mathbf{y}^{\mathrm{T}}\mathbf{W}\mathbf{y}-2\mathbf{y}^{\mathrm{T}}\mathbf{W}\mathbf{X}\boldsymbol{\beta}-2\boldsymbol{\eta}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{y}+2\boldsymbol{\beta}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\boldsymbol{\eta}+\boldsymbol{\beta}^{\mathrm{T}}\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\boldsymbol{\beta}+\boldsymbol{\eta}^{\mathrm{T}}\mathbf{S}^{\mathrm{T}}\mathbf{W}\mathbf{S}\boldsymbol{\eta}\end{aligned} \tag{10}$$

for obtaining estimator from $\boldsymbol{\beta}$, we used partial differential, $R(\boldsymbol{\beta},\boldsymbol{\eta})$ to $\boldsymbol{\beta}$ with $\partial R(\boldsymbol{\beta},\boldsymbol{\eta})/\partial\boldsymbol{\beta}$ is $\mathbf{0}$, so the result as follows :

$$\frac{\partial R(\boldsymbol{\beta},\boldsymbol{\eta})}{\partial\boldsymbol{\beta}} = \mathbf{0}$$

$$\frac{\partial\left\{(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta})^{\mathrm{T}}\mathbf{W}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta})\right\}}{\partial\boldsymbol{\beta}} = \mathbf{0}$$

$$-2\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y}+2\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}+2\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{0}$$

$$-\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y}+\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}+\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{0}$$

$$\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y}-\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}$$

so obtained

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left\{\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{y}-\mathbf{X}^{\mathrm{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}\right\} \tag{11}$$

With same procedure, $\boldsymbol{\eta}$ can be obtained based on partial differential, $R(\boldsymbol{\beta},\boldsymbol{\eta})$ where $R(\boldsymbol{\beta},\boldsymbol{\eta})$ is $\mathbf{0}$. So, it can be resulted as follows:

$$\frac{\partial R(\boldsymbol{\beta},\boldsymbol{\eta})}{\partial\boldsymbol{\eta}} = \mathbf{0}$$

$$\frac{\partial\left\{(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta})^{\mathbf{T}}\mathbf{W}(\mathbf{y}-\mathbf{X}\boldsymbol{\beta}-\mathbf{S}\boldsymbol{\eta})\right\}}{\partial\boldsymbol{\eta}}=\mathbf{0}$$

$$-2\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}+2\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}+2\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}=\mathbf{0}$$

$$-\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}+\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}+\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}=\mathbf{0}$$

$$\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\hat{\boldsymbol{\eta}}=\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}$$

so obtained

$$\hat{\boldsymbol{\eta}}=\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\left\{\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right\} \tag{12}$$

i. Equation (11) and (12) are not free from parameter. based on inference Statistical, an estimator should have satisfied sufficient criteria. So, $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\eta}}$ is determined without contain other parameters. In this case substitution procedure to derive $\hat{\boldsymbol{\beta}}$ that not include parameter, equation (12) is substituted equation (11)

$$\hat{\boldsymbol{\beta}}=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left[\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left\{\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\left\{\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right\}\right\}\right]$$

$$=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left[\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left\{\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right\}\right]$$

$$=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left[\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}+\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}\right]$$

$$\hat{\boldsymbol{\beta}}=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}+\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}$$

Furthermore, the terms containing parameters are grouped in one segment

$$\hat{\boldsymbol{\beta}}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X}\hat{\boldsymbol{\beta}}=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}$$

The parameters are divided into

$$\hat{\boldsymbol{\beta}}(\mathbf{I}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X})=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}$$

Then obtained

$$\hat{\boldsymbol{\beta}}=(\mathbf{I}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X})^{-1}\left(\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}\right)$$

If defined $\mathbf{M}=(\mathbf{I}-\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{X})^{-1}$ an estimator $\hat{\boldsymbol{\beta}}$ for the parametric component is obtained

$$\begin{aligned}\hat{\boldsymbol{\beta}}&=\mathbf{M}\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{y}-\mathbf{M}\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{y}\\&=\mathbf{M}\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left\{\mathbf{X}^{\mathbf{T}}-\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\right\}\mathbf{W}\mathbf{y}\\&=\mathbf{A}(\mathbf{K})\mathbf{y}\end{aligned} \tag{13}$$

with $\mathbf{A}(\mathbf{K})=\mathbf{M}\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{X}\right)^{-1}\left\{\mathbf{X}^{\mathbf{T}}-\mathbf{X}^{\mathbf{T}}\mathbf{W}\mathbf{S}\left(\mathbf{S}^{\mathbf{T}}\mathbf{W}\mathbf{S}\right)^{-1}\mathbf{S}^{\mathbf{T}}\right\}\mathbf{W}$

The theorem 2 part (i) has been proven.

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ii. To get $\hat{\boldsymbol{\eta}}$ that free from parameter substitute equation (11) into equation (12).

$$\hat{\boldsymbol{\eta}} = (\mathbf{S}^T\mathbf{WS})^{-1}\left[\mathbf{S}^T\mathbf{Wy} - \mathbf{S}^T\mathbf{WX}\left\{(\mathbf{X}^T\mathbf{WX})^{-1}\{\mathbf{X}^T\mathbf{Wy} - \mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}}\}\right\}\right]$$

$$= (\mathbf{S}^T\mathbf{WS})^{-1}\left[\mathbf{S}^T\mathbf{Wy} - \mathbf{S}^T\mathbf{WX}\left\{(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy} - (\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}}\right\}\right]$$

$$= (\mathbf{S}^T\mathbf{WS})^{-1}\left[\mathbf{S}^T\mathbf{Wy} - \mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy} + \mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}}\right]$$

$$\hat{\boldsymbol{\eta}} = (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{Wy} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy} + (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}}$$

Furthermore, the terms containing parameters are grouped in one segment

$$\hat{\boldsymbol{\eta}} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}} = (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{Wy} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy}$$

The parameters are divided into

$$\hat{\boldsymbol{\eta}}(\mathbf{J} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS} = (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{Wy} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy})$$

Then obtained

$$\hat{\boldsymbol{\eta}} = (\mathbf{I} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS})^{-1}\left((\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{Wy} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy}\right)$$

$$\hat{\boldsymbol{\eta}} = (\mathbf{S}^T\mathbf{WS})^{-1}\left[\mathbf{S}^T\mathbf{Wy} - \mathbf{S}^T\mathbf{WX}\left\{(\mathbf{X}^T\mathbf{WX})^{-1}\{\mathbf{X}^T\mathbf{Wy} - \mathbf{X}^T\mathbf{WS}\hat{\boldsymbol{\eta}}\}\right\}\right]$$

If defined $\mathbf{N} = (\mathbf{I} - (\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{WS})^{-1}$ an estimator $\hat{\boldsymbol{\eta}}$ for the nonparametric component is obtained

$$\hat{\boldsymbol{\eta}} = \mathbf{N}(\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{Wy} - \mathbf{N}(\mathbf{S}^T\mathbf{WS})^{-1}\mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\mathbf{Wy}$$

$$= \mathbf{N}(\mathbf{S}^T\mathbf{WS})^{-1}\left\{\mathbf{S}^T - \mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\right\}\mathbf{Wy} \tag{14}$$

$$= \mathbf{B}(\mathbf{K})\mathbf{y}$$

with $\mathbf{B}(\mathbf{K}) = \mathbf{N}(\mathbf{S}^T\mathbf{WS})^{-1}\left\{\mathbf{S}^T - \mathbf{S}^T\mathbf{WX}(\mathbf{X}^T\mathbf{WX})^{-1}\mathbf{X}^T\right\}\mathbf{W}$

The theorem 2 part (ii) has been proven.

iii. After resulting parametric and nonparametric components, for semiparametric estimation, Fourier series for longitudinal data. In semiparametric regression is given as follows:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\eta} + \boldsymbol{\varepsilon} \quad \varepsilon_{i,j} \sim IIN(0, \sigma^2)$$

$$E(\mathbf{y}) = E(\mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\eta} + \boldsymbol{\varepsilon})$$

$$E(\mathbf{y}) = E(\mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\eta}) + E(\boldsymbol{\varepsilon})$$

$$E(\mathbf{y}) = E(\mathbf{X}\boldsymbol{\beta} + \mathbf{S}\boldsymbol{\eta})$$

so that
$$\begin{aligned}\hat{\mathbf{y}} &= \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{S}\hat{\boldsymbol{\eta}} \\ &= \mathbf{XA(K)y} + \mathbf{SB(K)y} \\ &= (\mathbf{XA(K)} + \mathbf{SB(K)})\mathbf{y} \\ &= \mathbf{C(K)y}\end{aligned} \tag{15}$$

with $\mathbf{C(K)} = \mathbf{XA(K)} + \mathbf{SB(K)}$ is a hat matrix
The theorem 2 part (iii) has been proven.

3. Weighted matrix and the selection

Then the W matrix form used to estimate parameters $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$ are as follows:

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{W}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \mathbf{W}_n \end{bmatrix}$$

WLS is used because in longitudinal data observations that have correlated error. Furthermore [9], suggested some weighting matrices that can be used in parameter estimation for longitudinal data as follows:

I. $W_i = N^{-1} I_{n_i}$, subject $i = 1,2,3,\ldots,\mathrm{n}$ so that every measurement is treated the same

$W = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_n \end{bmatrix}$ subject component, with $W_i = \frac{1}{n} I_{n_i}$ so that $W_i = \frac{1}{N}\begin{bmatrix} I_1 & 0 & \cdots & 0 \\ 0 & I_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{n_i} \end{bmatrix}$ observation component

II. $W_i = (nn_i)^{-1} I_{n_i}$, subject $i = 1,2,3,\ldots,\mathrm{n}$ so that every measurement in the subject is treated the same

$W = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_n \end{bmatrix}$ subject component, with $W_i = \frac{1}{nn_i} I_{n_i}$ so that

$$W_i = \frac{1}{nn_i}\begin{bmatrix} I_1 & 0 & \cdots & 0 \\ 0 & I_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{n_i} \end{bmatrix}$$

observation component.

III. $W_i = V^{-1}$ where $V^{-1} = \text{var}(y_i)$, subject $i = 1,2,3,\ldots,n$ so taking into account the variance between subjects in the calculation.

$$W = \begin{bmatrix} W_1 & 0 & \cdots & 0 \\ 0 & W_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_n \end{bmatrix} = \begin{bmatrix} V_1^{-1} & 0 & \cdots & 0 \\ 0 & V_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & V_n^{-1} \end{bmatrix} \text{ with } V_i^{-1} = \begin{bmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \cdots & \sigma_{1n_i}^2 \\ \sigma_{21}^2 & \sigma_{22}^2 & \cdots & \sigma_{2n_i}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1}^2 & \sigma_{n2}^2 & \cdots & \sigma_{nn_i}^2 \end{bmatrix}$$

The W matrix is used in the WLS optimization to get the parameter estimator $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$. WLS optimization is done by minimizing error from the semiparametric regression, with it the Fourier series approach. While it measured based on k optimal which gives the smallest MSE value. Optimal oscillation parameter is chosen according Consideration of k will result in a high coefficient of determination or R^2.

4. Conclusions

Given a semiparametric regression model with Fourier series estimator for longitudinal data such as equation (3). The estimator has a random error that normal distributed with mean 0 and variance σ^2, estimator form parametric component in equation (13), nonparametric component in equation (14) and semiparametric regression component in equation (15). We obtain $\hat{\boldsymbol{\beta}} = \mathbf{M}(\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}\left\{\mathbf{X}^T - \mathbf{X}^T\mathbf{W}\mathbf{S}(\mathbf{S}^T\mathbf{W}\mathbf{S})^{-1}\mathbf{S}^T\right\}\mathbf{W}\mathbf{y}$ as parametric component parameter estimator, $\hat{\boldsymbol{\eta}} = \mathbf{N}(\mathbf{S}^T\mathbf{W}\mathbf{S})^{-1}\left\{\mathbf{S}^T - \mathbf{S}^T\mathbf{W}\mathbf{X}(\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}\mathbf{X}^T\right\}\mathbf{W}\mathbf{y}$ as nonparametric component parameter estimator, so we get parameter estimator for semiparametric regression in longitudinal data $\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}} + \mathbf{S}\hat{\boldsymbol{\eta}} = \mathbf{XA(K)y} + \mathbf{SB(K)y} = \mathbf{C(K)y}$. W is the weighting matrix with the forms of the weights $W_i = N^{-1}I_{n_i}$, $W_i = (nn_i)^{-1}I_{n_i}$, $W_i = V^{-1}$ based on [11], with the weights selection the cousin estimator according to the optimal oscillation parameter k with the smallest MSE and GCV value. A good model can be measured from the biggest determination coefficient value so that each type of weight can be applied in the application.

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Semiparametric regression based on fourier series for longitudinal data with Weighted Lest Square (WLS) optimization

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